PART II

NEGATION

A propositional question contains a demand that we should either acknowledge the truth of a thought, or reject it as false. In order that we may meet this demand correctly, two things are requisite: first, the wording of the question must enable us to recognize without any doubt the thought that is referred to; secondly, this thought must not belong to fiction. I always assume in what follows that these | conditions are fulfilled. The answer to a question[9] is an assertion based upon a judgement; this is so equally whether the answer is affirmative or negative.

Here, however, a difficulty arises. If a thought has being by being true, then the expression 'false thought' is just as self-contradictory, as 'thought that has no being'. In that case the expression 'the thought: three is greater than five' is an empty one; and accordingly in science it must not be used at all – except between quotation-marks. In that case we may not say 'that three is greater five is false'; for the grammatical subject is empty.

But can we not at least ask if something is true? In a question we can distinguish between the demand for a judgement and the special content of the question, the point as to which we must judge. In what follows I shall call this special content simply the content of the question, or the sense of the corresponding interrogative sentence. Now has the interrogative sentence

'Is 3 greater than 5?'

a sense, if the being of a thought consists in its being true? If not, the question cannot have a thought as its content; and one is inclined to say that the interrogative sentence has no sense at all. But this surely comes about because we see the falsity at once. Has the interrogative sentence

'Is $(21/20)^{100}$ greater than $\sqrt[10]{10^{21}}$?'

got a sense? If we had worked out that the answer must be affirmative,

---

[9]Here and in what follows I always mean a propositional question when I just write 'question'.

we could accept the interrogative sentence as making sense, for it would have a thought as its sense. But what if the answer had to be negative? In that case, on our supposition, we should have no thought that was the sense of the question. But surely the interrogative sentence must have some sense or other, if it is to contain a question at all. And are we really not asking for something in this sentence? May we not be wanting to get an answer to it? In that case, it depends on the answer whether we are to suppose that the question has a thought as its content. But it must be already possible to grasp the sense of the interrogative sentence before answering the question; for otherwise no answer would be possible at all. So that which we can grasp as the sense of the interrogative sentence before answering the question – and only this can properly be called the sense of the interrogative sentence – cannot be a thought, if the being of a thought consists in being true. 'But is it not a truth that the Sun is bigger than the Moon? And does not the being of a truth just consist in its being true? Must we not therefore recognize after all that the sense of the interrogative sentence:

'Is the Sun bigger than the Moon?'

is a truth, a thought whose being consists in its being true?' No! Truth cannot go along with the sense of an interrogative sentence; that would contradict the very nature of a question. The content of a question is 145   that as to which we must judge. | Consequently truth cannot be counted as going along with the content of the question. When I raise the question whether the Sun is bigger than the Moon, I am seeing the sense of the interrogative sentence

'Is the Sun bigger than the Moon?'

Now if this sense were a thought whose being consisted in its being true, then I should at the same time see that this sense was true,. Grasping the sense would at the same time be an act of judging; and the utterance of the interrogative sentence would at the same time be an assertion, and so an answer to the question. But in an interrogative sentence neither the truth nor the falsity of the sense may be asserted. Hence an interrogative sentence has not as its sense something whose being consists in its being true. The very nature of a question demands a separation between the acts of grasping a sense and of judging. And since the sense of an interrogative sentence is always also inherent in the assertoric sentence that gives an answer to the question, this separation must be carried out for assertoric sentences too. It is a matter of what we take the word 'thought' to mean. In any case, we need a short term for what can be the sense of an interrogative

sentence. I call this a thought. If we use language this way, not all thoughts are true. The being of a thought thus does not consist in its being true. We must recognize that there are thoughts in this sense, since we use questions in scientific work; for the investigator must sometimes content himself with raising a question, until he is able to answer it. In raising the question he is grasping a thought. Thus I may also say: The investigator must sometimes content himself with grasping a thought. This is anyhow already a step towards the goal, even if it is not yet a judgement. There must, then, be thoughts, in the sense I have assigned to the word. Thoughts that perhaps turn out later on to be false have a justifiable use in science, and must not be treated as having no being. Consider indirect proof; here knowledge of the truth is attained precisely through our grasping a false thought. The teacher says 'Suppose *a* were not equal to *b*.' A beginner at once thinks 'What nonsense! I can see that *a is* equal to *b*'; he is confusing the senselessness of a sentence with the falsity of the thought expressed in it.

Of course we cannot infer anything from a false thought; but the false thought may be part of a true thought, from which something can be inferred. The thought contained in the sentence:

> 'If the accused was in Rome at the time of the deed, he did not commit the murder'[10]

may be acknowledged to be true by someone who does not know if the accused was in Rome at the time of the deed nor if he committed the murder. Of the two component thoughts contained in the whole, neither the antecedent nor the consequent is being uttered assertively when the whole | is presented as true. We have then only a single act of judgement, but three thoughts, viz. the whole thought, the antecedent, and the consequent. If one of the clauses were senseless, the whole would be senseless. From this we see what a difference it makes whether a sentence is senseless or on the contrary expresses a false thought. Now for thoughts consisting of an antecedent and a consequent there obtains the law that, without prejudice to the truth, the opposite of the antecedent may become the consequent, and the opposite of the consequent the antecedent. In English this procedure is called *contraposition*.

According to this law, we may pass from the proposition

146

---

[10]Here we must suppose that these words by themselves do not contain the thought in its entirety; that we must gather from the circumstances in which they are uttered how to supplement them so as to get a complete thought.

'If $(21/20)^{100}$ is greater than $\sqrt[10]{10^{21}}$, then $(21/20)^{1000}$ is greater than 10^{21}'

to the proposition

'If $(21/20)^{1000}$ is not greater than 10^{21}, then $(21/20)^{100}$ is not greater than $\sqrt[10]{10^{21}}$'.

And such transitions are important for indirect proofs, which would otherwise not be possible.

Now if the first complex thought has a true antecedent, viz, $(21/20)^{100}$ *is greater than* $\sqrt[10]{10^{21}}$, then the second complex thought has a false consequent, viz. $(21/20)^{100}$ *is not greater than* $\sqrt[10]{10^{21}}$. So anybody that admits the legitimacy of our transition from *modus ponens* to *modus tollens* must acknowledge that even a false thought has being; for otherwise either only the consequent would be left in the *modus ponens* or only the antecedent in the *modus tollens*; and one of these would likewise be abolished as a nonentity.

The being of a thought may also be taken to lie in the possibility of different thinkers' grasping the thought as one and the same thought. In that case the fact that a thought had no being would consist in several thinkers' each associating with the sentence a sense of his own; this sense would in that case be a content of his particular consciousness, so that there would be no *common* sense that could be grasped by several people. Now is a false thought a thought that in this sense has no being? In that case investigators who had discussed among themselves whether bovine tuberculosis is communicable to men, and had finally agreed that such communicability did not exist, would be in the same position as people who had used in conversation the expression 'this rainbow', and now came to see that they had not been designating anything by these words, since what each of them had had was a phenomenon of which he himself was the owner. The investigators would have to realize that they had been deceived by a false appearance; for the presupposition that could alone have made all their activity and talk reasonable would have turned out not to be fulfilled; they would not have been giving the question that they discussed a sense common to all of them.

But it must be possible to put a question to which the true answer is negative. The content of such a question is, in my terminology, a thought. It must be possible for several people who hear the same interrogative sentence to grasp the same sense and recognize the falsity of it. Trial by jury would assuredly be a silly arrangement if it could not be assumed that each of the jurymen could understand the question

at issue in the same sense. So the sense of an interrogative sentence, even when the question has to be answered in the negative, is something that can be grasped by several people.

What else would follow if the truth of a thought consisted in the possibility of its being grasped by several people as one and the same thing, whereas a sentence that expressed something false had no sense common to several people?

If a thought is true and is a complex of thoughts of which one is false, then the whole thought could be grasped by several people as one and the same thing, but the false component thought could not, Such a case may occur. E.g. it may be that the following assertion is justifiably made before a jury: 'If the accused was in Rome at the time of the deed, he did not commit the murder'; and it may be false that the accused was in Rome at the time of the deed. In that case the jurymen could grasp the same thought when they heard the sentence 'If the accused was in Rome at the time of the deed, he did not commit the murder,' whereas each of them would associate a sense of his own with the *if-*clause. Is this possible? Can a thought that is present to all the jurymen as one and the same thing have a part that is not common to all of them? If the whole needs no owner, no part of it needs an owner.

So a false thought is not a thought that has no being – not even if we take 'being' to mean 'not needing an owner.' A false thought must be admitted, not indeed as true, but as sometimes indispensable: first, as the sense of an interrogative sentence; secondly, as part of a hypothetical thought-complex; thirdly, in negation. It must be possible to negate a false thought, and for this I need the thought; I cannot negate what is not there. And by negation I cannot transform something that needs me as its owner into something of which I am not the owner, and which can be grasped by several people as one and the same thing.

Now is negation of a thought to be regarded as dissolution of the thought into its component parts? By their negative verdict the jury can in no way alter the make-up of the thought that the question presented to them expresses. The thought is true or false quite independently of their giving a right or a wrong verdict in regard to it. And if it is false it is still a thought. If after the jury's verdict there is no thought at all, but only fragments of thought, then the same was already the case before the verdict; in what looked like a question, the jury were not presented with any thought at all, but only with fragments of thought; they had nothing to pass a verdict on.

Our act of judgement can in no way alter the make-up of a thought. We can only acknowledge what is there. A true thought cannot | be   148 affected by our act of judgement. In the sentence that expresses the

N

thought we can insert a 'not'; and the sentence we thus get does not contain a non-thought (as I have shown) but may be quite justifiably used as antecedent or consequent in a hypothetical sentence complex. Only, since it is false, it may not be uttered assertively. But this procedure does not touch the original thought in any way; it remains true as before.

Can we affect a false thought somehow by negating it? We cannot do this either; for a false thought is still a thought and may occur as a component part of a true thought. The sentence

'3 is greater than 5 ,

uttered non-assertively, has a false sense; if we insert a 'not,' we get

'3 is not greater than 5',

a sentence that may be uttered assertively. There is no trace here of a dissolution of the thought, a separation of its parts.

How, indeed, could a thought be dissolved? How could the inter-connection of its parts be split up? The world of thoughts has a model in the world of sentences, expressions, words, signs. To the structure of the thought there corresponds the compounding of words into a sentence; and here the order is in general not indifferent. To the dissolution or destruction of the thought there must accordingly correspond a tearing apart of the words, such as happens, e.g., if a sentence written on paper is cut up with scissors, so that on each scrap of paper there stands the expression for part of a thought. These scraps can then be shuffled at will or carried away by the wind; the connection is dissolved, the original order can no longer be recognized. Is this what happens when we negate a thought? No! The thought would un-doubtedly survive even this execution of it in effigy. What we do is to insert the word 'not,' and, apart from this, leave the word-order unaltered. The original wording can still be recognized; the order may not be altered at will. Is this dissolution, separation? Quite the reverse! it results in a firmly-built structure.

Consideration of the law *duplex negatio affirmat* makes it specially plain to see that negation has no separating or dissolving effect. I start with the sentence

'The Schneekoppe is higher than the Brocken'.

By putting in a 'not' I get:

'The Schneekoppe is not higher than the Brocken'.

(Both sentences are supposed to be uttered non-assertively.) A second negation would produce something like the sentence

> 'It is not true that the Schneekoppe is not higher than the Brocken'.

We already know that the first negation cannot effect any dissolution of the thought; but all the same let us suppose for once that after the first negation we had only | fragments of a thought. We should then have to suppose that the second negation could put these fragments together again. Negation would thus be like a sword that could heal on again the limbs it had cut off. But there the greatest care would be wanted. The parts of the thought have lost all connection and inter-relation on account of its being negated the first time. So by carelessly employing the healing power of negation, we might easily get the sentence:

149

> 'The Brocken is higher than the Schneekoppe'.

No non-thought is turned into a thought by negation, just as no thought is turned into a non-thought by negation.

A sentence with the word 'not' in its predicate may, like any other, express a thought that can be made into the content of a question; and this, like any propositional question, leaves open our decision as to the answer.

What then are these objects, which negation is supposed to separate? Not parts of sentences: equally, not parts of a thought. Things in the outside world? They do not bother about our negating. Mental images in the interior world of the person who negates? But then how does the juryman know which of his images he ought to separate in given circumstances? The question put before him does not indicate any to him. It may evoke images in him. But the images evoked in the jurymen's inner worlds are different; and in that case each juryman would perform his own act of separation in his own inner world, and this would not be a verdict.

It thus appears impossible to state what really is dissolved, split up, or separated by the act of negation.

With the belief that negation has a dissolving or separating power there hangs together the view that a negative thought is less useful than an affirmative one. But still it cannot be regarded as wholly useless. Consider the inference:

> 'If the accused was not in Berlin at the time of the murder, he did not commit the murder; now the accused was not in Berlin at the time of the murder; therefore he did not commit the murder',

and compare it with the inference:

> 'If the accused was in Rome at the time of the murder, he did
> not commit the murder; now the accused was in Rome at the
> time of the murder; therefore he did not commit the murder'.

Both inferences proceed in the same form, and there is not the least
ground in the nature of the .case for our distinguishing between
negative and affirmative premises when we are expressing the law of
inference here involved. People speak of affirmative and negative
judgements; even Kant does so. Translated into my terminology, this
would be a distinction between affirmative and negative thoughts. For
logic at any rate such a distinction is wholly unnecessary; its ground
must be sought outside logic. I know of no logical principle whose
150 verbal expression makes it necessary, or | even preferable, to use these
terms.[11] In any science in which it is a question of conformity to laws,
the thing that we must always ask is: What technical expressions are
necessary or at least useful, in order to give precise expression to the
laws of this science? What does not stand this test cometh of evil.[12]

What is more, it is by no means easy to state what is a negative
judgement (thought). Consider the sentences 'Christ is immortal',
'Christ lives for ever', 'Christ is not immortal', 'Christ is mortal', 'Christ
does not live for ever'. Now which of the thoughts we have here is
affirmative, which negative?

We usually suppose that negation extends to the whole thought
when 'not' is attached to the verb of the predicate. But sometimes the
negative word grammatically forms part of the subject, as in the
sentence 'no man lives to be more than a hundred.' A negation may
occur anywhere in a sentence without making the thought indubitably
negative. We see what tricky questions the expression 'negative judge-
ment (thought)' may lead to. The result may be endless disputes, carried
on with the greatest subtlety, and nevertheless essentially sterile.
Accordingly I am in favour of dropping the distinction between
negative and affirmative judgements or thoughts until such time as we
have a criterion enabling us to distinguish with certainty in any given
case between a negative and an affirmative judgement. When we have

---

[11]Accordingly, in my essay 'Thoughts', *Beiträge zur Philosophie des deutschen Idealismus*
I, p. 58 [p. 351 above], I likewise made no use of the expression 'negative thought.' The
distinction between negative and affirmative thoughts would only have confused the
matter. At no point would there have been occasion to assert something about affirmative
thoughts, excluding negative ones, or to assert something about negative thoughts,
excluding affirmative ones.

[12][An apparent allusion to Matthew v. 37! (*Tr.*)]

such a criterion we shall also see what benefit may be expected from this distinction. For the present I still doubt whether this will be achieved. The criterion cannot be derived from language; for languages are unreliable on logical questions. It is indeed not the least of the logician's tasks to indicate the pitfalls laid by language in the way of the thinker.

After refuting errors, it may be useful to trace the sources from which they have flowed. One source, I think, in this case is the desire to give definitions of the concepts one means to employ. It is certainly praiseworthy to try to make clear to oneself as far as possible the sense one associates with a word. But here we must not forget that not everything can be defined. If we insist at any price on defining what is essentially indefinable, we readily fasten upon inessential accessories, and thus start the inquiry on a wrong track at the very outset. And this is certainly what has happened to many people, who have tried to explain what a judgement is and so have | hit upon compositeness.[13] 151 The judgement is composed of parts that have a certain order, an interconnection, stand in mutual relations; but for what whole we do not get this?

There is another mistake associated with this one: viz. the view that the judging subject sets up the connection or order of the parts in the act of judging and thereby brings the judgement into existence. Here the act of grasping a thought and the acknowledgement of its truth are not kept separate. In many cases, of course, one of these acts follows so directly upon the other that they seem to fuse into one act; but not so in all cases. Years of laborious investigations may come between

---

[13]We are probably best in accord with ordinary usage if we take a judgement to be an act of judging, as a leap is an act of leaping. Of course this leaves the kernel of the difficulty uncracked; it now lies in the word 'judging'. Judging, we may say, is acknowledging the truth of something; what is acknowledged to be true can only be thought. The original kernel now seems to have cracked in two; one part of it lies in the word 'thought' and the other in the word 'true'. Here, for sure, we must stop. The impossibility of an infinite regress in definition is something we must be prepared for in advance.

If a judgement is an act, it happens at a certain time and thereafter belongs to the past. With an act there also belongs an agent, and we do not know the act completely if we do not know the agent. In that case, we cannot speak of a synthetic judgement in the usual sense. If we call it a synthetic judgement that through two points only one straight line passes, then we are taking 'judgement' to mean, not an act performed by a definite man at a definite time, but something timelessly true, even if its being true is not acknowledged by any human being. If we call this sort of thing a truth, then we may perhaps with advantage say 'synthetic truth' instead of 'synthetic judgement'. If we do nevertheless prefer the expression 'synthetic judgement', we must leave out of consideration the sense of the verb 'to judge'.

grasping a thought and acknowledging its truth. It is obvious that here the act of judging did not make the thought or set its parts in order; for the thought was already there. But even the act of grasping a thought is not a production of the thought, is not an act of setting its parts in order; for the thought was already true, and so was already there with its parts in order, before it was grasped. A traveller who crosses a mountain-range does not thereby make the mountain-range; no more does the judging subject make a thought by acknowledging its truth. If he did, the same thought could not be acknowledged as true by one man yesterday and another man to-day; indeed, the same man could not recognize the same thought as true at different times – unless we supposed that the existence of the thought was an intermittent one.

If someone thinks it within his power to produce by an act of judgement that which, in judging, he acknowledges to be true, by setting up an interconnection, an order, among its parts; then it is easy for him to credit himself also with the power of destroying it. As destruction is opposed to construction, to setting up order and
152   interconnection, so also negating seems to be opposed to judging; | and people easily come to suppose that the interconnection is broken up by the act of negation just as it is built up by the act of judgement. Thus judging and negating look like a pair of polar opposites, which, being a pair, are coordinate; a pair comparable, e.g., to oxidation and reduction in chemistry. But when once we see that no interconnection is set up by our judging; that the parts of the thought were already in their order before our judging; then everything appears in a different light. It must be pointed out yet once more that to grasp a thought is not yet to judge; that we may express a thought in a sentence without asserting its truth; that a negative word may be contained in the predicate of a sentence, in which case the sense of this word is part of the sense of the sentence, part of the thought; that by inserting a 'not' in the predicate of a sentence meant to be uttered non-assertively, we get a sentence that expresses a thought, as the original one did. If we call such a transition, from a thought to its opposite, negating the thought, then negating in this sense is not coordinate with judging, and may not be regarded as the polar opposite of judging; for what matters in judging is always the truth, whereas we may pass from a thought to its opposite without asking which is true. To exclude misunderstanding, let it be further observed that this transition occurs in the consciousness of a thinker, whereas the thoughts that are the *termini a quo* and *ad quem* of the transition were already in being before it occurred; so that this psychical event makes no difference to the make-up and the mutual relations of the thoughts.

Perhaps the act of negating, which maintains a questionable existence as the polar opposite of judging, is a chimerical construction, formed by a fusion of the act of judging with the negation that I have acknowledged as a possible component of a thought, and to which there corresponds in language the word 'not' as part of the predicate – a chimerical construction, because these parts are quite different in kind. The act of judging is a psychical process, and as such it needs a judging subject as its owner; negation on the other hand is part of a thought, and as such, like the thought itself, it needs no owner, must not be regarded as a content of a consciousness. And yet it is not quite incomprehensible how there can arise at least the illusion of such a chimerical construction. Language has no special word or syllable to express assertion; assertive force is supplied by the form of the assertoric sentence, which is specially well-marked in the predicate. On the other hand the word 'not' stands in intimate connection with the predicate and may be regarded as part of it. Thus a connection may seem to be formed between the word 'not' and the assertoric force in language that answers to the act of judging.

But it is a nuisance to distinguish between the two ways of negating. Really my only aim in introducing the polar opposite of judging was to accommodate myself to a way of thinking that is foreign to me. I now return to my previous | way of speaking. What I have just been 153 designating as the polar opposite of judging I will now regard as a second way of judging – without thereby admitting that there is such a second way. I shall thus be comprising both polar opposites under the common term 'judging'; this may be done, for polar opposites certainly do belong together. The question will then have to be put as follows:

Are there two different ways of judging, of which one is used for the affirmative, and the other for the negative, answer to a question? Or is judging the same act in both cases? Does negating go along with judging? Or is negation part of the thought that underlies the act of judging? Does judging consist, even in the case of a negative answer to a question, in acknowledging the truth of a thought? In that case the thought will not be the one directly contained in the question, but the opposite of this.

Let the question run, e.g., as follows: 'Did the accused intentionally set fire to his house?' How can the answer take the form of an assertoric sentence, if it turns out to be negative? If there is a special way of judging for when we deny, we must correspondingly have a special form of assertion. I may, e.g., say in this case 'it is false that...' and lay it down that this must always have assertoric force attached to it. Thus the answer will run something like this: 'It is false that the

accused intentionally set fire to his house.' If on the other hand there is only one way of judging, we shall say assertorically: 'The accused did not intentionally set fire to his house.' And here we shall be presenting as something true the opposite thought to the expressed in the question. The word 'not' here belongs with the expression of this thought. I now refer back to the two inferences I compared together just now. The second premise of the first inference was the negative answer to the question 'was the accused in Berlin at the time of the murder?' – in fact, the answer that we fixed upon in case there is only one way of judging. The thought contained in this premise is contained in the *if*-clause of the first premise, but there it is uttered non-assertively. The second premise of the second inference was the affirmative answer to the question. 'Was the accused in Rome at the time of the murder?' These inferences proceed on the same principle, which is in good agreement with the view that judging is the same act whether the answer to a question is affirmative or negative. If on the other hand we had to recognize a special way of judging for the negative case – and correspondingly, in the realm of words and sentences, a special form of assertion – the matter would be otherwise. The first premise of the first inference would run as before:

> 'If the accused was not in Berlin at the time of the murder, he did not commit the murder'.

Here we could not say 'If it is false that the accused was in Berlin at the time of the murder'; for we have laid it down that to the words 'it is false that' assertoric force must always be attached; but in acknowledg-
154 ing the truth of this first premise we are not | acknowledging the truth either of its antecedent or if its consequent. The second premise on the other hand must now run: 'It is false that the accused was in Berlin at the time of the murder'; for being a premise it must be uttered assertively. The inference now cannot be performed in the same way as before; for the thought in the second premise no longer coincides with the antecedent of the first premise; it is now the thought that the accused *was* in Berlin at the time of the murder. If nevertheless we want to allow that the inference is valid, we are thereby acknowledging that the second premise contains the thought that the accused was *not* in Berlin at the time of the murder. This involves separating negation from the act of judging, extracting it from the sense of 'it is false that...', and uniting negation with the thought.

Thus the assumption of two different ways of judging must be rejected. But what hangs on this decision? It might perhaps be regarded as valueless, if it did not effect an economy of logical primitives and

their expressions in language. On the assumption of two ways of judging we need:

1. assertoric force for affirmatives;
2. assertoric force for negatives, e.g. inseparably attached to the word 'false';
3. a negating word like 'not' in sentence uttered non-assertorically.

If on the other hand we assume only a single way of judging, we only need:

1. assertoric force;
2. a negating word.

Such economy always shows that analysis has been pushed further, which leads to a clearer insight. There hangs together with this an economy as regards a principle of inference; with our decision we can make do with one where otherwise we need two. If we *can* make do with one way of judging, then we *must*; and in that case we cannot assign to one way of judging the function of setting up order and connection, and to another, the function of dissolving this.

Thus for every thought there is a contradictory[14] thought; we acknowledge the falsity of a thought by admitting the truth of its contradictory. The sentence that expresses the contradictory thought is formed from the expression of the original thought by means of a negative word.

The negative word or syllable often seems to be more closely united to part of the sentence, e.g. the predicate. This may lead us to think that what is negated is the content, not of the whole sentence, but just of this part. We may call a man uncelebrated and thereby indicate the falsity of the thought that he is celebrated. This may be regarded as the negative | answer to the question 'is the man celebrated?'; and hence we may see that we are not here just negating the sense of a word. It is incorrect to say: 'Because the negative syllable is combined with part of the sentence, the sense of the whole sentence is not negated.' On the contrary: it is by combining the negative syllable with a part of the sentence that we do negate the content of the whole sentence. That is to say: in this way we get a sentence in which there is a thought contradicting the one in the original sentence.

I do not intend by this to dispute that negation is sometimes restricted just to a part of the whole thought.

155

[14]We could also say 'an opposite thought'.

If one thought contradicts another, then from a sentence whose sense is the one it is easy to construct a sentence expressing the other. Consequently the thought that contradicts another thought appears as made up of that thought and negation. (I do not mean by this, the act of denial.) But the words 'made up of,' 'consist of,' 'component,' 'part' may lead to our looking at it the wrong way. If we choose to speak of parts in this connection, all the same these parts are not mutually independent in the way that we are elsewhere used to find when we have parts of a whole. The thought does not, by its make-up, stand in any need of completion; it is self-sufficient. Negation on the other hand needs to be completed by a thought. The two components, if we choose to employ this expression, are quite different in kind and contribute quite differently towards the formation of the whole. One completes, the other is completed. And it is by this completion that the whole is kept together. To bring out in language the need for completion, we may write 'the negation of...', where the blank after 'of' indicates where the completing expression is to be inserted. For the relation of completing, in the realm of thoughts and their parts, has something similar corresponding to it in the realm of sentences and their parts. (The preposition 'of', ['*von*'], followed by a substantive can also be replaced [in German] by the genitive of the substantive; this may as a rule be more idiomatic, but does not lend itself so well to the purpose of expressing the part that needs completion.) An example may make it even clearer what I have here in mind. The thought that contradicts the thought:

$$(21/20)^{100} \text{ is equal to } \sqrt[10]{10^{21}}$$

is the thought:

$$(21/20)^{100} \text{ is not equal to } \sqrt[10]{10^{21}}.$$

We may also put this as follows:

'The thought:

$$(21/20)^{100} \text{ is not equal to } \sqrt[10]{10^{21}}$$

is the negation of the thought:

$$(21/20)^{100} \text{ is equal to } \sqrt[10]{10^{21}}\text{'.} \mid$$

156   In the last expression (after the penultimate 'is') we can see how the thought is made up of a part that needs completion and a part that completes it. From now on I shall use the word 'negation' (except, e.g.,

within quotation marks) always with the definite article. The definite article '*the*' in the expression

'*the* negation of the thought that 3 is greater than 5'

shows that this expression is meant to designate a definite single thing. This single thing is in our case a thought. The definite article makes the whole expression into a singular name, a proxy for a proper name.

The negation of a thought is itself a thought, and can again be used to complete *the negation*.[15] If I use, in order to complete *the negation*,[15] the negation of the thought that $(21/20)^{100}$ is equal to $^{10}\sqrt{10^{21}}$, what I get is:

the negation of the negation of the thought that $(21/20)^{100}$ is equal to $^{10}\sqrt{10^{21}}$.

This is again a thought. Designations of thoughts with such a structure are got according to the pattern:

'the negation of the negation of *A*',

where '*A*' takes the place of the designation of a thought. Such a designation is to be regarded as directly composed of the parts:

'the negation of ——'
and 'the negation of *A*'.

But it may also be regarded as made up of the parts:

'the negation of the negation of ——'
and: '*A*'.

Here I have first combined the middle part with the part that stands to the left of it and then combined the result with the part '*A*' that stands to the right of it; whereas originally the middle part was combined with '*A*,' and the designation so got, viz.

'the negation of *A*',

was combined with what stood to the left of it

'the negation of ——'.

---

[15][I.e. to complete the thought-component whose verbal expression is '*the negation* (*of*)...', so as to get a complete thought; just as, in the realm of language, we get a complete designation of a thought by inserting a designation of a thought in the blank of 'the negation of——'. (The italics in the text are ours, not Frege's.) (*Tr.*)]

The two different ways of regarding the designation have answering to them two ways of regarding the structure of the thought designated.[16]
If we compare the designations:

'the negation of the negation of: $(21/20)^{100}$ is equal to $^{10}\sqrt{10^{21}}$' and 'the negation of the negation of: 5 is greater than 3'

we recognize a common constituent:

'the negation of the negation of ——': |

157   this designates a part common to the two thoughts – a thought-component that stands in need of completion. In each of our two cases, it is completed by means of a thought: in the first case, the thought that $(21/20)^{100}$ is equal to $^{10}\sqrt{10^{21}}$; in the second case, the thought that 5 is greater than 3. The result of this completion is in either case a thought. This common component, which stands in need of completion, may be called double negation. This example shows how something that needs completion can be amalgamated with something that needs completion to form something that needs completion. Here we are presented with a singular case; we have something – the negation of... – amalgamated with itself. Here, of course, metaphors derived from the corporeal realm fail us; for a body cannot be amalgamated with itself so that the result is something different from it. But then neither do bodies need completion, in the sense I intend here. Congruent bodies *can* be put together; and in the realm of designations we have congruence in our present case. Now what corresponds to congruent designations is one and the same thing in the realm of designata.
Metaphorical expressions, if used cautiously, may after all help towards an elucidation. I compare that which needs completion to a wrapping, e.g. a coat, which cannot stand upright by itself; in order to do that, it must be wrapped round somebody. The man whom it is wrapped round may put on another wrapping, e.g. a cloak. The two wrappings unite to form a single wrapping. There are thus two possible ways of looking at the matter; we may say either that a man who already wore a coat was now dressed up in a second wrapping, a cloak, or, that his clothing consists of two wrappings – coat and cloak. These ways of looking at it have absolutely equal justification. The additional wrapping always combines with the one already there to form a new wrapping. Of course we must never forget in this connection that

[16][*Bezeichnenden* is here surely a misprint for *bezeichneten* or *zu bezeichnenden*. (*Tr.*)]

dressing up and putting things together are processes in time, whereas what corresponds to this in the realm of thoughts is timeless.

If *A* is a thought not belonging to fiction, the negation of *A* likewise does not belong to fiction. In that case, of the two thoughts: *A* and the negation of *A*: there is always one and only one that is true. Likewise, of the two thoughts: the negation of *A*, and the negation of the negation of *A*: there is always one and only one that is true. Now the negation of *A* is either true or not true. In the first case, neither *A* nor the negation of the negation of *A* is true. In the second case, both *A* and the negation of the negation of *A* are true. Thus of the two thoughts: *A*, and the negation of the negation of *A*: either both are true or neither is. I may express this as follows:

Wrapping up a thought in double negation does not alter its truth-value.